TOUCHING AND COUNTING

Prerequisites
- Counting
- Numeral identification 1-9

The Teaching Process
When demonstrating the touching/counting process:
1. The one is touched at the top while counting: "One."

2. The two is touched at the beginning and the end of the numeral while counting: "One, two."

3. The three is touched at the beginning, middle and end of the numeral while counting: "One, two, three."

4. The four is touched and counted from top to bottom on the down strokes while counting: "One, two, three, four."

5. The five is touched and counted in the order pictured: "One, two, three, four, five." The fourth Touchpoint may be referred to as the "belly button" to help students remember it.

Key Points To Remember
- *Single Touchpoints are touched and counted one time and double Touchpoints (dots and circles) are touched and counted twice.
- *Students touch with pencil points, count aloud as they touch the Touchpoints, and should consistently follow the correct touching/counting pattern.
6. The six begins the use of dots with circles. The encircled dots should be touched and counted twice, whenever they appear. Six is touched and counted from top to bottom: "One-two, three-four, five-six."

7. The seven is touched and counted from top to bottom: "One-two, three-four, five-six," followed by the single dot: "seven." The single Touchpoint can be thought of as the nose. Teachers sometimes tell young or remedial students to, "punch him in the nose" to help them remember the final Touchpoint.

8. The eight is touched and counted from left to right: "One-two, three-four, five-six, seven-eight." Tell the young or remedial students that the eight looks like a robot. Count his head first, and then his body.

9. The nine is touched and counted from top to bottom: "One-two, three-four, five-six, seven-eight," followed by the single dot: "nine." Again, the single Touchpoint can be thought of as the nose.

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**Key Points To Remember**

- *First graders will learn the touching/counting pattern in approximately one week, second graders will require two or three days, and third graders should be proficient in one class period.*
- *Students should use numbers with Touchpoints printed on the problems provided and should never be asked to draw them.*
Directions. Begin at the top of the column of numbers and use a pencil to touch and count the touchpoints in the correct order. Write the answer. Say the problem and answer quietly.

\[
\begin{align*}
0 &+ 2 \quad 1 &+ 2 \quad 2 &+ 2 \quad 3 &+ 2 \quad 4 &+ 2 \\
2 &+ 2 \quad 3 &+ 2 \quad 2 &+ 2 \quad 3 &+ 2 \quad 4 &+ 2 \\
5 &+ 2 \quad 6 &+ 2 \quad 7 &+ 2 \quad 8 &+ 2 \quad 9 &+ 2 \\
2 &+ 2 \quad 2 &+ 2 \quad 2 &+ 2 \quad 2 &+ 2 \quad 2 &+ 2 \\
6 &+ 2 \quad 3 &+ 2 \quad 8 &+ 2 \quad 9 &+ 2 \quad 2 &+ 2 \\
\end{align*}
\]

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\[
\begin{align*}
7 + 2 &= \_\_ \\
1 + 2 &= \_\_ \\
5 + 2 &= \_\_ \\
4 + 2 &= \_\_
\end{align*}
\]
Directions. Work the following problems using regrouping when it is necessary.

```
  2256
+ 5979
  8235

  1972
+ 7234
  9206

  1639
+ 3197
  4836

  1798
+ 4256
  6054

  2765
+ 4236
  7001

  5356
+ 2784
  8140

  1819
+ 2458
  4277

  4372
+ 2649
  7021

☆ Puzzler ☆

  1534
+ 6852
   8386

  3678
+ 2863
  6541

  2578
+ 1469
  4047

  5128
+ 3294
  8422
```

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Adding Four Columns with & without Regrouping 261A
Directions: Ring the sign before beginning each problem, and decide if you will be adding or subtracting. Then solve the problem, and write the answer.

\[
\begin{array}{cccccc}
12 & 9 & 16 & 14 & 9 \\
-6 & +3 & -8 & -7 & +9 \\
\hline
7 & 11 & 7 & 10 & 18 \\
+5 & -5 & +4 & -5 & -9 \\
\hline
4 & 0 & 12 & 10 & 14 \\
+2 & -0 & -5 & -8 & -6 \\
\hline
17 & 16 & 5 & 8 & 6 \\
-9 & -7 & +4 & +4 & -3 \\
\end{array}
\]
**Directions:** Start on the side with the arrow. The arrow is on the right side. Subtract the ones column, and write the answer. Then subtract the tens column, and write the answer.

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<table>
<thead>
<tr>
<th>Tens</th>
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<tbody>
<tr>
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**Puzzler**

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<td>-12</td>
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<td></td>
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</tbody>
</table>
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Directions: Solve the following problems through the thousands (TH) column, regrouping when necessary. Cross out the regrouping lines you don't need.

\[
\begin{align*}
\text{TH H T O} & \quad \text{TH H T O} & \quad \text{TH H T O} & \quad \text{TH H T O} \\
9,638 & \quad 8,632 & \quad 7,418 & \quad 6,000 \\
-2,772 & \quad -5,817 & \quad -4,804 & \quad -4,927 \\
\hline
6,866 & \quad 2,815 & \quad 2,614 & \quad 1,073 \\
\end{align*}
\]

★ Puzzler ★

Subtract these numbers horizontally.

\[
\begin{align*}
92 - 71 &= \_\_\_\_\_ \\
342 - 285 &= \_\_\_\_\_ \\
623 - 362 &= \_\_\_\_\_ \\
453 - 207 &= \_\_\_\_\_ \\
\end{align*}
\]
Directions. Touch the touchpoints on the top number as you sequence count by two, then write the answer. No guessing! Say the problem and answer quietly.

<table>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
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<td>×2</td>
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<table>
<thead>
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<th>7</th>
<th>8</th>
<th>9</th>
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<tr>
<td>×2</td>
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<th>7</th>
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<tbody>
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☆ Puzzler ☆

8 × 2 = ___  9 × 2 = ___
5 × 2 = ___  4 × 2 = ___
Directions. Read each problem. Trace the lines below the picture as you sequence count. Multiply the problem. Rewrite the answer on the lines. Work the puzzler.

1. There are 2 balls in each box.
   There are 4 boxes.
   Touch the dots and count by 2.
   How many balls are there total?  
   Answer  \[ \underline{8} \]  \underline{balls}  

2. There are 2 balls in each box.
   There are 6 boxes.
   Touch the dots and count by 2.
   How many balls are there in all?  
   Answer  \[ \underline{12} \]  \underline{balls}  

3. There are 2 balls in each box.
   There are 5 boxes.
   Touch the dots and count by 2.
   How many balls are there altogether?  
   Answer  \[ \underline{10} \]  \underline{balls}  

☆ Puzzler ☆
Sequence count by 2 to 20.

\[ \underline{2} \underline{4} \underline{6} \underline{8} \underline{10} \underline{12} \underline{14} \underline{16} \underline{18} \underline{20} \]
Directions. If you know the answer, write it down. If not, touch the touchpoints on the first number as you sequence count by the second number, then write the answer.

\[ 4 \times 5 = \]
\[ 6 \times 3 = \]
\[ 7 \times 2 = \]
\[ 9 \times 1 = \]
\[ 8 \times 5 = \]
\[ 3 \times 4 = \]
\[ 2 \times 2 = \]
\[ 7 \times 0 = \]
\[ 9 \times 3 = \]
\[ 4 \times 3 = \]
\[ 5 \times 5 = \]
\[ 9 \times 5 = \]
\[ 7 \times 1 = \]
\[ 2 \times 4 = \]
\[ 4 \times 2 = \]
\[ 6 \times 4 = \]
\[ 8 \times 4 = \]
\[ 7 \times 5 = \]
\[ 6 \times 4 = \]
\[ 8 \times 3 = \]
**Directions.** Multiply the numbers in the ones column. Write the number to be regrouped in the box, then write the remaining number under the ones column. Multiply the tens column, add the regrouped number and write the answer.

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**Puzzler**

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<td>× 2</td>
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</tbody>
</table>
Directions. Multiply by the number in the ones column, then the number in the tens column, regrouping as necessary. Write the regrouped number in the correct box.

```
567
x  93
1701
51030
52731

456
x  74
3412
34184
0

669
x  85
5095
56115
0

888
x  66
5328
53288
0

765
x  56
4610
43660
0

578
x  47
2626
26266
0
```
Directions. Read each problem. Sequence count by the divisor up to the dividend making tally marks as you count. Count the tally marks to solve the problem. Rewrite the answer on the lines. Work the puzzler.

1. There are 6 pears total. 2 boys divide the pears equally. How many pears does each boy receive?

Answer 3 pears

2. There are 9 pears total. 3 boys divide the pears equally. How many pears does each boy receive?

Answer

3. There are 6 pears total. Draw circles to show dividing the pears into sets of 2. If each circled set is given to different girls, how many girls would receive pears?

Answer

☆ Puzzler ☆ The following are division words:
- divide = Take apart or separate into groups
- divisor = The number being used to divide
- dividend = The number or quantity to be divided
- quotient = The answer in a division problem
- remainder = The number left that is less than the divisor

Write the dividend and divisor.

\[ \underline{2 \div 6} \]

dividend = [ ]
divisor = [ ]
Directions. Sequence count by the divisor and make tally marks in the box as you say each number. When you cannot continue sequence counting, make a dot outside of the box for each remaining number. Write the answer and remainder as shown.

\[ \begin{array}{cccc}
2)7 & 2)11 & 2)17 & 2)13 \\
3)7 & 3)10 & 3)16 & 3)22 \\
4)21 & 4)33 & 4)37 & 4)17 \\
5)26 & 5)36 & 5)41 & 5)21 \\
\end{array} \]

\[ \begin{array}{cc}
5 \div 2 = & 29 \div 4 = \\
13 \div 3 = & 16 \div 5 = \\
\end{array} \]
**Directions:** No guessing! If you know the answer to each problem, write it down. If not, count by the divisor up to the dividend making a tally mark in the box as you say each number. Count the tally marks and write the answer. Say the problem and answer quietly.

<table>
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<tr>
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<th>36</th>
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<th>14</th>
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<tr>
<td>9</td>
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**Puzzler**

16 ÷ 2 = ___  
24 ÷ 3 = ___  
28 ÷ 4 = ___  
30 ÷ 5 = ___
Directions: Solve the problems using short and long division. Write the answer and the remainder.

Short Division | Long Division
--- | ---
4 | 4
53 | 53
-12 | -
33 | 33
-32 | -

Short Division | Long Division
--- | ---
5 | 5
321 | -
-
-

8 | 8
734 | 734
- | -
- | -

© TouchMath Upper Grades Multiplication & Division Kit 2
Three-Digit Short and Long Division
Addition Statement

I touch the largest number, say its name, and continue counting.
Arrow Statement

I start on the side with the arrow. The arrow is on the right side.
Regrouping Statement

I must borrow or regroup, if I cannot continue to count all the Touchpoints.
Subtraction Statement

I touch the top number, say its name, and count backwards.
Regrouping Statement

I must borrow or regroup, if I cannot continue to count all the Touchpoints.
Multiplication Statement

I sequence count by □ while touching the Touchpoints on the □.
Division Statement

I sequence count by _______ and get as close to _______ as I can without going over _______.